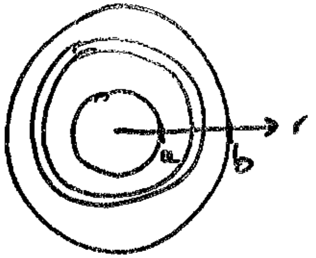
Survie dans un igloo

(1)



Bilan d'énergie durant dt sur une portion comprise entre r et $r+dr$.

$$d^2U = \delta U(r, t+dt) - \delta U(r,t)$$

$$d^2U = \left(u(r,t+dt) - u(r,t)\right) d\tau = \frac{\partial u(r,t)}{\partial t} dt\, d\tau$$

$$d^2U = \rho c \frac{\partial T(r,t)}{\partial t} dt\, d\tau.$$

Causes de variation.

$$\vec{j}_{th}(r,t) \cdot \vec{dS}_r \qquad \vec{j}_{th}(r+dr,t) \cdot \vec{dS}_{r+dr}$$

$$d^2U = \delta Q(r,t) + \delta Q(r+dr,t) + \delta Q_{source}.$$

$$d^2U = \left[\phi_{th}(r,t)\,dt - \phi_{th}(r+dr,t)\right]dt = -\frac{\partial \phi_{th}(r,t)}{\partial r} dr\, dt$$

$$\phi_{th}(r,t) = \iint \vec{j}_{th}(r,t) \cdot \vec{dS}_r = \vec{j}_{th}(r,t) \cdot \vec{e}_r \times 4\pi r^2$$

$$d^2U = -\frac{\partial}{\partial r}\left(\vec{j}_{th}(r,t) \cdot \vec{e}_r \times r^2\right) 4\pi\, dr\, dt.$$

Loi de Fourier : $\vec{j}_{th} = -\lambda\, \vec{grad}\, T = -\lambda \frac{\partial T(r,t)}{\partial r} \vec{e}_r$

$$d^2U = +\lambda \frac{\partial}{\partial r}\left[r^2 \frac{\partial T}{\partial r}\right] \times 4\pi\, dr\, dt.$$

$$4\pi r^2 \rho c \frac{\partial T}{\partial t} dr\, dt = 4\pi\lambda \frac{\partial}{\partial r}\left[r^2\left(\frac{\partial T}{\partial r}\right)\right] dr\, dt$$

$$\frac{\lambda}{\rho c} \frac{1}{r^2} \frac{\partial}{\partial r}\left[r^2\left(\frac{\partial T}{\partial r}\right)\right] = \frac{\partial T}{\partial t}$$

formulaire

$$\Delta T = \frac{1}{r} \frac{\partial^2}{\partial r^2}(rT) = \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{\partial}{\partial r}(rT)\right]$$

$$\Delta T = \frac{1}{r}\frac{\partial}{\partial r}\left[T + r\frac{\partial T}{\partial r}\right] = \frac{1}{r}\left[2\frac{\partial T}{\partial r} + r\frac{\partial^2 T}{\partial r^2}\right]$$

$$\Delta T = \frac{2}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(\frac{\partial T}{\partial r}\right)\right] = \frac{1}{r^2}\left[2r\left(\frac{\partial T}{\partial r}\right) + r^2\left(\frac{\partial^2 T}{\partial r^2}\right)\right]$$

$$= \frac{2}{r}\left(\frac{\partial T}{\partial r}\right) + \left(\frac{\partial^2 T}{\partial r^2}\right) = \Delta T$$

$$\boxed{\frac{\lambda}{\rho c}\Delta T = \frac{\partial T}{\partial t}}$$

(2) En régime stationnaire, $\Delta T = 0$

$$\frac{d}{dr}\left[r^2\left(\frac{dT}{dr}\right)\right] = 0 \Rightarrow r^2\left(\frac{dT}{dr}\right) = A$$

$$\frac{dT}{dr} = \frac{A}{r^2} \Rightarrow T(r) = -\frac{A}{r} + B$$

CL : $T(a) = T_1 = -\frac{A}{a} + B$

$T(b) = T_2 = -\frac{A}{b} + B$

$$T_1 - T_2 = -A\left(\frac{1}{a}-\frac{1}{b}\right) = -A\left(\frac{b-a}{ab}\right)$$

$$A = \frac{(T_1-T_2)ab}{a-b}$$

$$B = T_1 + \frac{A}{a} = T_1 + \frac{(T_1-T_2)ab}{a(a-b)}$$

$$B = T_1 + \frac{(T_1-T_2)\,b}{(a-b)} = \frac{T_1(a-\cancel{b}) + (\cancel{T_1}-T_2)b}{(a-b)}$$

$$B = \frac{T_1 a - T_2 b}{a-b}$$

$$T(r) = -\frac{(T_1-T_2)ab}{(a-b)}\,\frac{1}{r} + \frac{T_1 a - T_2 b}{a-b}$$

$$\phi_{th}(r,t) = \iint \vec{j}_{th}(r,t)\cdot\vec{dS} = -\lambda\frac{dT}{dr}4\pi r^2$$

$$\phi_{th}(r,t) = -4\pi\lambda A = -\frac{4\pi\lambda(T_1-T_2)ab}{(a-b)}$$

Rq : ϕ indep de r.

$$R_{th} = \frac{T_1-T_2}{\phi_{th}} = \frac{b-a}{4\pi\lambda ab}$$

(3) Igloo → hémisphère.

hyp : Pas de transfert thermique vers le sol.

$$\phi_{igloo} = \frac{2\pi\lambda(T_1-T_2)ab}{b-a} = \frac{T_1-T_2}{R_{igloo}}$$

$$R_{igloo} = \frac{b-a}{2\pi\lambda ab}$$

$$\phi_{igloo} = P = \frac{T_1-T_2}{R_{igloo}} = \frac{2\pi\lambda ab(T_1-T_2)}{b-a}$$

$$b = a+e \Rightarrow P = \frac{2\pi\lambda(a+e)a(T_1-T_2)}{\cancel{a}+e-\cancel{a}}$$

$$Pe = 2\pi\lambda a(T_1-T_2)(a+e)$$

$$(P - 2\pi\lambda a(T_1-T_2))e = 2\pi\lambda a^2(T_1-T_2)$$

$$e = \frac{a}{\dfrac{P}{2\pi\lambda a(T_1-T_2)} - 1} = \frac{1}{\dfrac{100}{2\pi\times 5.10^{-2}\times 1\times 30} - 1}$$

$e = 46$ cm